

Semester Two Examination, 2022

Question/Answer booklet

MATHEMATICS
METHODS
UNITS 1&2

**SOLUTIONS**

Section Two:
Calculator-assumed

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| WA student number: In figures |  |  |  |  |  |  |  |  |  |  |

 In words

 Your name

|  |  |
| --- | --- |
| Number of additionalanswer booklets used(if applicable): |  |

## Time allowed for this section

Reading time before commencing work: ten minutes

Working time: one hundred minutes

## Materials required/recommended for this section

***To be provided by the supervisor***

This Question/Answer booklet

Formula sheet (retained from Section One)

***To be provided by the candidate***

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener,
correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators, which can include scientific, graphic and Computer Algebra System (CAS) calculators, are permitted in this ATAR course examination

## Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

## Structure of this paper

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Section | Number ofquestionsavailable | Number ofquestions tobe answered | Workingtime(minutes) | Marksavailable | Percentageofexamination |
| Section One:Calculator-free | 7 | 7 | 50 | 53 | 35 |
| Section Two:Calculator-assumed | 12 | 12 | 100 | 96 | 65 |
|  |  | **Total** | 100 |

## Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen.
Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Section Two: Calculator-assumed 65% (96 Marks)

This section has**twelve** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 8 (8 marks)

An online store offers different art prints, of which are impressionist, are pop art and the remainder contemporary.

(a) Determine the number of different selections that can be made

(i) when different prints are bought from the store.

 (2 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ indicates correct use of combination notationü correct number of selections |

(ii) when different contemporary prints are bought from the store. (2 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ indicates correct use of combination notationü correct number of selections |

(b) A random selection of different prints sold by the store is made. Determine the probability that

(i) none of the prints in the selection are pop art. (3 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ indicates correct method for numeratorü indicates correct method for denominatorü correct probability |

(ii) at least one of the prints in the selection is pop art. (1 mark)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ correct probability |

Question 9 (8 marks)

In an industrial process to create hydrogen, an engineer observed that the higher the power density W/cm2, the shorter the cathode lifetime in hours, so that when and when .

The relationship between the variables is of the form , where and are constants.

(a) Determine the value of and the value of . (3 marks)

|  |
| --- |
| Solution |
| Solving simultaneously with CAS gives and . |
| Specific behaviours |
| ✓ uses points to form two equationsü correct value of ü correct value of  |

(b) If the process is only possible for , state the corresponding range of .

 (2 marks)

|  |
| --- |
| Solution |
| Hence range of is . |
| Specific behaviours |
| ✓ obtains lower or upper boundü correct domain as inequality |

(c) Graph the relationship on the axes below over the domain . (3 marks)

|  |
| --- |
| Solution |
| See graph |
| Specific behaviours |
| ✓ correct endpointsü evidence of using at least two other pointsü smooth curve |



Question 10 (9 marks)

Clinical records for sports injuries classified the type of sport involved as either team or individual. The records show that of injuries were the result of team sport, and that after initial treatment, of team sport and of individual sport related injuries required one or more follow up visits to the clinic.

Determine the probability that a randomly chosen sports injury

(a) was sustained through individual sport and required one or more follow up visits.

 (2 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ shows correct use of probability ruleü correct probability |

(b) required no follow up visit. (3 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ indicates correct probability for ü indicates correct probability for ü correct probability |

(c) was sustained through individual sport or required one or more follow up visits. (2 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ shows correct use of probability rulesü correct probability |

(d) was sustained through team sport, given that the patient was known to have required one or more follow up visits. (2 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ shows correct use of conditional probability ruleü correct probability |

Question 11 (8 marks)

(a) A large outdoor amphitheatre has seats in the first row, in the second row and so on, where each row has two more seats than the previous row. There are rows of seats in the amphitheatre.

(i) Determine the number of seats in the last row of the amphitheatre. (2 marks)

|  |
| --- |
| Solution |
|  seats. |
| Specific behaviours |
| ✓ indicates first term and common difference of sequenceü correct number of seats |

(ii) Determine the total number of seats in the last rows of the amphitheatre.

 (3 marks)

|  |
| --- |
| Solution |
| Hence in last rows there are seats. |
| Specific behaviours |
| ✓ sum of all rowsü sum of first rowsü correct number of seats in last rows |

(b) The sum of the first and second terms of a geometric series is , and the sum of the second and third terms of the series is . Determine the sum of the first three terms of the series. (3 marks)

|  |
| --- |
| Solution |
| Let first term and common ratio so thatHence sum of first three terms is . |
| Specific behaviours |
| ✓ expressions using and for both sumsü obtains correct ratioü correct sum of first three terms |

Question 12 (8 marks)

A function is defined by and the graph of is shown below.



(a) Draw the chord between the points and on the curve that have
-coordinates and respectively and determine the slope of this chord. (3 marks)

|  |
| --- |
| Solution |
| or |
| Specific behaviours |
| ✓ draws chord with rulerü calculates -coordinates of pointsü calculates gradient |

The point with -coordinate lies on the curve between and , where .

(b) Use the difference quotient to calculate the slope of chord when

(i) . (2 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ uses correct values of and ü correct slope, to at least dp |

(ii) . (1 mark)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ü correct slope, to at least dp |

(c) Show use of the difference quotient to determine the slope of tangent to the curve at that is correct to decimal places. (2 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ chooses where ü correct slope |

Question 13 (8 marks)

(a) Express as an exact radian measure and hence determine the length of an arc of a circle that subtends an angle of at the centre of a circle of radius cm. (2 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ angle in radiansü correct arc length |

(b) Determine the area of a segment of a circle that subtends an angle of at the centre of a circle of radius cm. (2 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ substitutes correctlyü correct area of segment |

(c) The lengths of the sides of a triangle with an area of cm2 are in the ratio . Determine the length of the shortest side of the triangle. (4 marks)

|  |
| --- |
| Solution |
| Let angle opposite longest side be so that(Other possible angles are or )Then using area formula with sides of and :Hence shortest side is . |
| Specific behaviours |
| ✓ shows use of cosine rule to obtain angleü shows use of area formula with variableü solves for variableü correct length of required side |

Question 14 (9 marks)

Let be the sum of the first terms of an arithmetic sequence with first term and common difference .

The sum of the first terms of the sequence is .

(a) Show that . (2 marks)

|  |
| --- |
| Solution |
| Using sum of AP formula: |
| Specific behaviours |
| ✓ substitutes correctly into sum of AP formulaü shows at least one correct simplification step |

The sum of the first terms of the sequence is .

(b) Determine the sum of the first terms of the sequence. (3 marks)

|  |
| --- |
| Solution |
| Equation using second sum, as in (a):Solving simultaneously and (CAS) gives and .Hence . |
| Specific behaviours |
| ✓ forms second equationü solves equations for ü calculates correct sum |

The sum can be expressed in the form , where and are constants.

(c) Determine the value of , the value of and then use calculus to show that is the maximum sum of the sequence. (4 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ values of and ü correctly differentiatesü correctly solves derivative equal to for ü justifies maximum nature using either a sign table or the second derivative test. |

Question 15 (7 marks)

The graph of is shown below, where is a positive constant.



|  |
| --- |
| Solution (a)(ii) |
| See graph labelled  |
| Specific behaviours |
| ✓ smooth curve through ü translation units down and draw the asymptote |

|  |
| --- |
| Solution (a)(i) |
| See graph labelled  |
| Specific behaviours |
| ✓ smooth curve through ü translation units right |

(a) On the axes above sketch and label the graphs of

(i) . (2 marks)

(ii) . (2 marks)

(b) Given that the curve passes through the point , solve for .

 (3 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ forms equation for ; ü correct value of ü correct value of  |

Question 16 (8 marks)

For two events and , and . Determine the value of the constant in each of the following cases:

(a) and are mutually exclusive. (1 mark)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ correct value of  |

(b) . (2 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ formulates correct equation for ü correct value of  |

(c) . (2 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ formulates correct equation for ü correct value of  |

(d) . (3 marks)

|  |
| --- |
| Solution |
| Since then and are independent. |
| Specific behaviours |
| ✓ indicates independenceü formulates correct equation for ü correct value of  |

Question 17 (8 marks)

The length of a cuboid is twice its width, and the sum of its height, width and length is cm. Let the width of the cuboid be cm.

(a) Show that the volume of the cuboid is cm3. (2 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ correct expression for heightü correct factored expression for volume |

(b) Use a method involving differentiation to determine the height of the cuboid that maximises its volume. (3 marks)

|  |
| --- |
| Solution |
| Hence when .Height for maximum volume is cm. |
| Specific behaviours |
| ✓ differentiates correctlyü equates derivative to and solves for ü states correct height |

(c) Determine the maximum possible total surface area of the cuboid. (3 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ expression for TSAü determines optimum value of ü states correct maximum TSA |

Question 18 (9 marks)

(a) Determine the global minimum and maximum values of the function when the domain of is restricted to . (2 marks)

|  |
| --- |
| Solution |
| Using CAS, has a local maximum at , and .Hence global minimum value of function is and global maximum is  |
| Specific behaviours |
| ✓ correct global minimumü correct global maximum |

(b) Determine the discriminant of and use it to explain how many roots the function has. (2 marks)

|  |
| --- |
| Solution |
| Hence has roots as the discriminant is greater than . |
| Specific behaviours |
| ✓ correct value of ü number of roots, with reasoning |

(c) The graph of is symmetrical about the line and passes through the points and . Determine the roots of the graph. (4 marks)

|  |
| --- |
| Solution |
| When .Using line of symmetryUsing :HenceUsing CAS, when . |
| Specific behaviours |
| ✓ obtains value of ü correct method usedü solves for ü correct roots |

Question 19 (7 marks)

A team of workers is using a pile driver to drive wooden poles 4 metres long into the ground. The first hit of the pile driver drives a pole 50 cm into the ground. The second hit drives the pole another 40 cm into the ground. The third hit drives the pole another 32 cm into the ground and successive distances driven by the pile driver form a geometric sequence.

(a) How much further will the fifth hit drive the pole into the ground? (1 mark)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ü correct solution |

(b) Determine the total distance the wooden pole has been driven into the ground after 12 hits of the pile driver. (2 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ uses any correct methodü correct solution |

(c) If the worker continued in this way for some time, what length of the wooden pole will always be left above ground? Justify your answer. (2 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ indicates correct long term steady stateü correct solution |

A second team of workers is also using a different pile driver to drive the same length poles into the ground. The first hit of this pile driver drives the pole 12 cm into the ground. The second hit drives the pole cm into the ground and the third hit drives the pole another 27 cm into the ground.

(d) Determine given that the successive distances form a geometric sequence. (2 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ uses any correct methodü correct solution |

Supplementary page

Question number: \_\_\_\_\_\_\_\_\_

Supplementary page

Question number: \_\_\_\_\_\_\_\_\_

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